九十八學年度第一學期微積分會考試題 (A 卷)

說明:
(1) 答題之前請先檢查所取得之試卷與答案卷是否正確。
(2) 測驗時間 110 分鐘。試卷加答案卷共計 5 頁。
(3) 試卷包括選擇題與填充題，總分共計 100 分，占學期成績之 30%。考卷成績將做為微積分獎給獎依據。
(4) 請先確實在答案卷填入相關個人資料。答題時請依題號空格作答，否則不予計分。

◎ 單選擇題 (單選十題，每題五分，共五十分，答錯不倒扣)

1. When using the $\varepsilon$ - $\delta$ definition to prove that $\lim_{x \to 0} (2x - x^2) = 0$ for $\varepsilon = 1$ the largest $\delta$ among the below answers is
   
   (A) $\frac{\sqrt{6}}{2}$;      (B) $\frac{\sqrt{6}}{2}$;      (C) $\frac{\sqrt{6}}{2}$;    (D) $\frac{6+\sqrt{6}}{2}$.

2. Let $F(x^3) = f(g(x^4))$, then which of following is true?
   
   (A) $F'(x) = 4x^3 f'(g(x^4))g'(x^4)$;           (B) $F'(x) = \frac{4x}{3} f'(g(x^4))g'(x^4)$;
   
   (C) $F'(x) = \frac{4}{3} x^{1/3} f'(g(x^{4/3}))g'(x^{4/3})$;        (D) $F'(x) = f'(g(x^4))g'(x^4)$.

3. Suppose $f$ is continuous on $(0, \infty)$ and, for $x > 0$, $\int_0^x f(t)dt = 1 + x$. Then $f(4) =$
   
   (A) $\frac{1}{4}$;      (B) $\frac{1}{2}$;          (C) 1;         (D) none of the above.

4. The value of $\lim_{x \to 0} \left( \frac{1}{\ln(x+1)} - \frac{x+1}{x} \right)$ is
   
   (A) 0;      (B) $-\frac{1}{2}$;    (C) $-1$;    (D) nonexistent.
5. Evaluate the limit \( \lim_{n \to \infty} \frac{1}{2n} \sum_{i=1}^{2n} \frac{1}{1 + \left( \frac{2i}{n} \right)^2} \) as a definite integral.

(A) \( \int_0^1 \frac{1}{1+2x^2} \, dx \);  
(B) \( \int_0^1 \frac{1}{2(1+2x^2)} \, dx \);  
(C) \( \int_0^1 \frac{1}{2(1+2x^2)} \, dx \);  
(D) \( \int_0^1 \frac{1}{2(1+2x^2)^2} \, dx \).

6. Evaluate the integral \( \int_0^1 x^5 e^{-x^2} \, dx \)

(A) \( \frac{1}{2} \);  
(B) \( \frac{1}{3} \);  
(C) \( 1 - \frac{1}{3} e^{-1} \);  
(D) \( \frac{1}{3} - \frac{2}{3} e^{-1} \).

7. For which positive real number \( b \) the average value of \( f(x) = 2 + 6x - 3x^2 \) on the interval \( [0, b] \) is largest?

(A) \( b = \frac{3 - \sqrt{5}}{2} \);  
(B) \( b = 1.5 \);  
(C) \( b = \frac{3 + \sqrt{5}}{2} \);  
(D) \( b = 0.000001 \).

8. Suppose \( f(\theta) > 0 \) for \( \theta \in [a, b] \) and \( f(\theta) \) is continuous on \( [a, b] \). Which of the following statement is always true about the arc length \( L \) of the polar curve \( r = f(\theta) \) for \( \theta \in [a, b] \)?

(A) \( L < \int_a^b f(\theta) \, d\theta \);  
(B) \( L = \int_a^b f(\theta) \, d\theta \);  
(C) \( L > \int_a^b f(\theta) \, d\theta \);  
(D) None of the above is always true.

9. To find the surface area of the solid of revolution formed by rotating \( y = x^3 \) over \( [0, 2] \) about the x-axis, we must evaluate

(A) \( \int_0^2 2\pi x \sqrt{1 + 9x^4} \, dx \);  
(B) \( \int_0^2 2\pi x^3 \sqrt{1 + 9x^4} \, dx \);  
(C) \( \int_0^2 2\pi x^3 \sqrt{1 + x^2} \, dx \);  
(D) \( \int_0^2 2\pi x^3 \sqrt{1 + x^2} \, dx \).
10. A plane flying horizontal at an altitude of 1 mile and a speed of 500 miles/hour passes directly over a radar station. Which of the following statement about the rate $r$ of change of the distance from the plane to the station when it is 3 miles away from the station is true.

(A) $1 < r < 500$;  
(B) $r = 500$;  
(C) $r > 500$;  
(D) $r = \frac{2\sqrt{2}}{3}$.

◎ 多選擇題 (多選五題，每題五分，共二十五分。答錯一個選項扣兩分，錯兩個選項以上不給分，分數不倒扣)

1. Which of the following statements are true for $f(x) = (\ln x^2) / x$?

(A) $f$ has the absolute maximum value $\frac{1}{e}$;  
(B) $f$ is concave downward on $(0,1)$;  
(C) The graph of $f$ has no inflection point;  
(D) $f$ is increasing on $(0,1)$.

2. Which of the following statements about the real function $f(x)$ on $(a,b)$ are true?

(A) If $f(x)$ has a local maximum at some point $c \in (a,b)$ then $f'(c)$ does not exist or $f''(c) = 0$.  
(B) If $f(x)$ is continuous on $(a,b)$ then $f(x)$ must have an absolute maximum on $(a,b)$.  
(C) If $f^2(x)$ is differentiable on $(a,b)$, then $f(x)$ must be differentiable on $(a,b)$.  
(D) If $f^n(x)$ exists on $(a,b)$, then $f'(x)$ is continuous on $(a,b)$.

3. Which of the following are true?

(A) $\lim_{x \to 0} \frac{\sin(x)}{x} = 1$;  
(B) $\lim_{x \to \infty} \frac{\sin(x)}{x} = 0$;  
(C) $\lim_{x \to 0} x \sin \left( \frac{1}{x} \right)$ does not exist;  
(D) $\lim_{x \to \infty} x \sin \left( \frac{1}{x} \right) = 1$.  

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4. Let \( y = 6x^3 + 5x - 3 \) represent a plane curve. Which of the following can not be the slope of a tangent line of the curve?

(A) 4; (B) 5; (C) 10; (D) 1.

5. Let \( f(x) \) be a real function such that \( f''(x) \) exists for all real numbers \( x \). Which of the following are the right expressions of \( f''(x) \)?

(A) \( f''(x) = \lim_{h \to 0} \frac{f(x + h) + f(x - h) - 2f(x)}{h^2} \);
(B) \( f''(x) = \lim_{h \to 0} \frac{f(x + h) + f(x - h) - 2f(x)}{2h} \);
(C) \( f''(x) = \lim_{h \to 0} \frac{f'(x + h) + f'(x - h) - 2f'(x)}{2h} \);
(D) \( f''(x) = \lim_{h \to 0} \frac{f'(x + h) - f'(x - h)}{2h} \).

◎ 填空題（五題，每題五分，共二十五分，答錯不倒扣）

1. \( y = \) (1) \( \) is the equation of the tangent line to the curve \( y = 2 - x^3 \) at the point (1,1).

2. \( \lim_{x \to \infty} \left( 1 - \frac{3}{x} + \frac{5}{x^2} \right)^x = \) (2) \( \).

3. If \( h(2) = 4 \) and \( h'(2) = -2 \), find \( \frac{d}{dx} \left( \frac{h(x)}{x} \right) \bigg|_{x=2} = \) (3) \( \).

4. \( \int_0^1 \frac{2x}{1 + x^2} \, dx = \) (4) \( \).

5. Find the value \( m = \) (5) \( \) such that the line \( y = mx \) and the curve \( y = x / (x^2 + 1) \) enclose a region of area \( -0.8 + \ln 5 \).