Calculus Exercise 15.1

3. (a) Use a Riemann sum with \( m = n = 2 \) to estimate the value of \( \int_{R} \sin(x+y) \, dA \), where \( R = [0, \pi] \times [0, \pi] \). Take the sample points to be lower left corners.

\[ \int_{R} \sin(x+y) \, dA \approx \sum_{i=1}^{2} \sum_{j=1}^{2} f(x_{ij}, y_{ij}) \Delta A \]
\[ = \{ f(0,0) + f(0,\pi/2) + f(\pi/2,0) + f(\pi/2,\pi/2) \} \Delta A \]
\[ = \{0+1+1+0\} \frac{\pi^2}{4} = \frac{\pi^2}{2} \approx 4.935 \]

3. (b) Use the Midpoint Rule to estimate the integral in part (a).

\[ \int_{R} \sin(x+y) \, dA \approx \sum_{i=1}^{2} \sum_{j=1}^{2} f(x_{ij}, y_{ij}) \Delta A \]
\[ = \{ f(\pi/4,\pi/4) + f(3\pi/4,\pi/4) + f(3\pi/4,3\pi/4) + f(\pi/4,3\pi/4) \} \Delta A \]
\[ = \{1+0+0+1\} \frac{\pi^2}{4} = 0 \]

11. Evaluate the double integral by first identifying it as the volume of a solid.
\[ \int_{R} 3 \, dA, \quad R = \{(x,y)| -2 \leq x \leq 2, 1 \leq y \leq 6 \} \]
\[ <\text{SOL}> z = 3 > 0, \text{ so we can interpret the integral as the volume of solid S whose height is 3 and above the rectangle [-2, 2] \times [1, 6].} \]
\[ \text{S is a rectangular solid, thus } \int_{R} 3 \, dA = 3 \times (4 \times 5) = 60 \]

13. Evaluate the double integral by first identifying it as the volume of a solid.
\[ \int_{R} (4-2y) \, dA, \quad R = [0, 1] \times [0, 1] \]
\[ <\text{SOL}> z = 4 - 2y > 0 \text{ in } R. \]
\[ \text{Thus the integral represents the volume of the rectangular solid which lies below } z = 4 - 2y \text{ and above the rectangle [0, 1] \times [0, 1].} \]
\[ \text{So } \int_{R} (4-2y) \, dA = 1 \times 1 \times 2 + \frac{1}{2} \times 1 \times 2 = 3 \]