一百零六學年度第二學期微積分會考試題 (A 卷)

說明:

(1) 答題之前請先檢查所取得之試卷與答案卷是否正確。

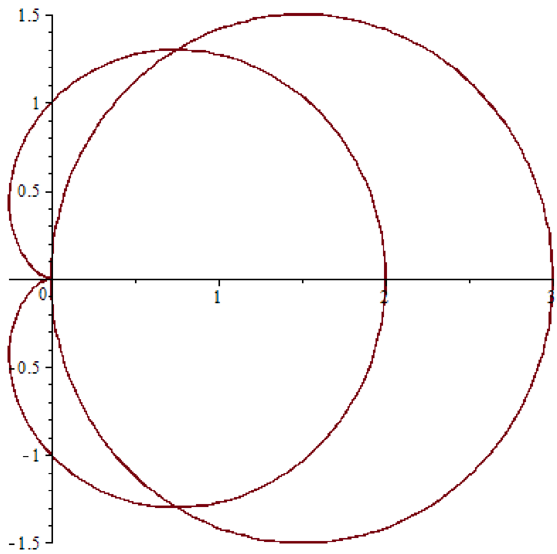
(2) 測驗時間 110 分鐘。試卷加答案卷、答案卡共計 6 頁。

(3) 試卷包括選擇題與填充題，總分共計 100 分，占學期成績之 30%。考卷成績將做為微積分獎給獎依據。

(4) 請先確實在答案卡與答案卷填入相關個人資料。答題時請依題號作答，否則不予計分。

◎ 單選擇題 (單選十題，每題五分，共五十分，答錯不倒扣)

1. The area of the region that lies inside the polar curve $r = 3\cos\theta$ and outside the polar curve $r = 1 + \cos\theta$ is



(A) $\pi + \sqrt{3}$; (B) π; (C) $\pi - \sqrt{3}$; (D) $\frac{1}{2}\pi$.

2. Which one is the equation for the **tangent line** to the curve

$$\gamma(t) = e^{-t}\cos t\ i + e^{-t}\sin t\ j + e^{-t}\ k \text{ at } (\mathbf{1},\mathbf{0},\mathbf{1})\ ?$$

(A) $x = 1 + t, y = t, z = 1 - t,\ t \in R$;

(B) $x = 1 - t, y = t, z = 1 - t,\ t \in R$;

(C) $x = t, y = 1 + t, z = 1 + t,\ t \in R$;

(D) $x = 1 - t, y = -t, z = 1 + t,\ t \in R$.

3. The **radius** of convergence for $\sum_{n=0}^{\infty} 3^n x^{2n}$ is

(A) $\frac{1}{3}$; (B) $\sqrt{3}$; (C) $\frac{1}{\sqrt{3}}$; (D) 3.

4. Which one of the following statements is **False**?

   (A) If the series $\sum_{n=1}^{\infty} a_n$ is convergent, then $\lim_{n\to\infty} a_n = 0$.

   (B) If $\lim_{n\to\infty} a_n = 0$, then $\lim_{n\to\infty} |a_n| = 0$.

   (C) The series $\sum_{n=1}^{\infty} n^{-\sin 2}$ is divergent.

   (D) If $\sum_{n=1}^{\infty} a_n = A$ and $\sum_{n=1}^{\infty} b_n = B$, then $\sum_{n=1}^{\infty} a_n b_n$=*AB*.

5. Let

$$f(x,y) = \begin{cases} \dfrac{x^2y^3}{x^4+y^4}, & \text{if } (x,y) \neq (0,0); \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$$

   Then, the **directional derivative** of f at $(0,0)$ in the direction of $\langle 1/\sqrt{2}, 1/\sqrt{2} \rangle$ is

   (A) 0; (B) $\sqrt{2}/4$; (C) $\sqrt{2}/2$; (D) $\sqrt{2}$.

6. The **maximum** value of $x^2 + y^2$ subject to the constraint $x^2 - 2x + y^2 - 4y = 0$ is

   (A) 10; (B) 20; (C) 30; (D) 40.

7. What is the value of the following limit

$$\lim_{(x,y)\to(1,1)} \frac{\ln(x)\ln(y)}{(\ln(x))^2 + (\ln(y))^2}.$$

   (A) 0; (B) $\frac{1}{2}$; (C) $-\frac{1}{2}$; (D) Does not exist.

8. The **area** of the part of the surface $z = xy$ that lies within the cylinder $x^2 + y^2 = 1$ and **above** the xy plane is $\frac{\pi}{3}\left(2\sqrt{2} + H\right)$. Then H is

   (A) 0; (B) 1; (C) -1; (D) -2.

9. Which value of the following formula is **different** from the others?

   (A) $\int_0^1 \int_{x^2-1}^{x-1} dydx$;

   (B) $\int_0^{-1} \int_{\sqrt{y+1}}^{y+1} dxdy$;

   (C) $\int_0^1 \int_{y+1}^{\sqrt{y+1}} dxdy$;

   (D) $\frac{1}{6}$.

10. Let C be the intersection of S_1 and S_2, where S_1, S_2 are level surfaces given by

$$S_1: xyz + 2yz + x^2z^2 = 1,\ S_2: x^2y^2z^2 + yz - x^2z = -1$$

Assume that the tangent line to C at $(1,0,1)$ is given by $\frac{x-1}{a} = \frac{y}{2} = \frac{z-1}{b}$. Then, $a+b$ equals

(A) -3; (B) -2; (C) -1; (D) 0.

◎ 多選擇題 (多選五題，每題五分，共二十五分。答錯一個選項扣兩分，錯兩個選項以上不給分，分數不倒扣)

11. Which of the following series **converge absolutely**?

(A) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n(n+1)}}$;

(B) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 2^n}{(2n+1)!}$;

(C) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(\ln n)^2}{n^2}$;

(D) $\sum_{n=1}^{\infty} \frac{\arctan n}{n^2}$.

12. Let $f(s,t)$ be a continuous function on $(-\infty,\infty)\times(-\infty,\infty)$ and $F(x,y) = \iint_{[0,x]\times[0,y]} f(s,t)\,dA$ for $(x,y) \in (0,\infty)\times(0,\infty)$. Which of the following statements are **True**?

(A) F is continuous.

(B) F is differentiable.

(C) $F_{xy} \neq F_{yx}$.

(D) If $f(s,t) = -f(t,s)$, then $F_x(x,x) + F_y(x,x) = 0$ for all $x > 0$.

13. What numbers of α and β with $\alpha \le \beta$ maximizes the value of the integral $\int_\alpha^\beta (6 - x - x^2)\,dx$?

(A) $\alpha = -3$; (B) $\alpha = -1$; (C) $\beta = 0$; (D) $\beta = 2$.

14. Suppose f is a function for which

$$\{(x,y) : f_x(x,y) = 0 \text{ and } f_y(x,y) = 0\} = \{(1,1),(1,-1),(-1,1),(-1,-1)\}.$$

Further suppose that for $a = -1,1$ and $b = -1,1$

$$f_{xx}(a,b) = ab, f_{yy}(a,b) = ab, f_{xy}(a,b) = a - b$$

Which of the following points are **local minimum**?

(A) $(1,1)$; (B) $(1,-1)$; (C) $(-1,1)$; (D) $(-1,-1)$.

15. Which of the following iterated integrals are **equal** to the integral

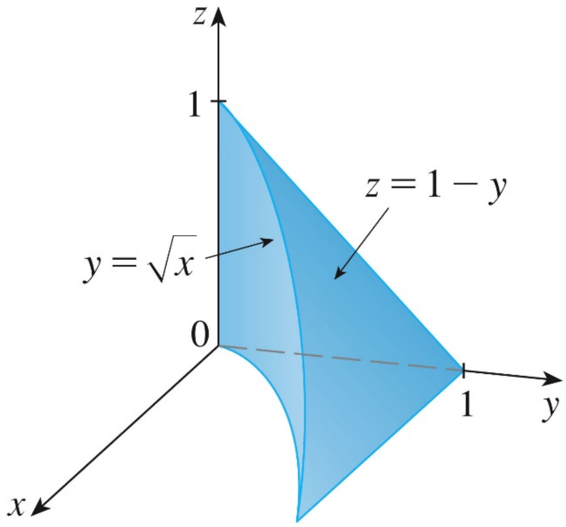
$$\int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} f(x,y,z)\,dzdydx.$$

(A) $\int_0^1 \int_{\sqrt{u}}^1 \int_0^{1-v} f(u,v,w)\,dwdvdu$;

(B) $\int_0^1 \int_0^{1-y} \int_0^{y^2} f(x,y,z)\,dxdzdy$;

(C) $\int_0^1 \int_0^{1-z} \int_0^{y^2} f(x,y,z)\,dxdydz$;

(D) $\int_0^1 \int_0^{(1-z)^2} \int_x^{1-z} f(x,y,z)\,dydxdz$.



◎ 填空題　(五題，每題五分，共二十五分，答錯不倒扣)

1. The **slope** of the tangent line to the polar curve $r = \frac{1}{\theta}$ at $\theta = \pi$ is ____(1)____.

2. The **interval of convergence** for $\sum_{n=1}^{\infty} \frac{(-1)^n}{n3^n} x^n$ is ____(2)____.

3. Let E be the trapezoidal region in the xy-plane with vertices $(1,0), (2,0), (0,-2)$ and $(0,-1)$. Consider the transformation T, defined by $u = x + y$ and $v = x - y$, from the xy-plane to the uv-plane. Let $T(E) = \{(u,v) | u = x + y, v = x - y \text{ and } (x,y) \in E\}$. Then the **area** of $T(E)$ is ____(3)____.

4. $\lim_{n\to\infty} n^{-2} \sum_{i=1}^{n} \sum_{j=1}^{n^2} \frac{1}{\sqrt{n^2+ni+j}} =$ ____(4)____.

5. The **volume** of the solid bounded by the planes $y = 0, z = 0, z = 1$ and the parabolic cylinder $y = 1 - x^2$ is ____(5)____.