一百零五學年度第二學期微積分會考試題 (A 卷)

說明:
(1)答題之前請先檢查所取得之試卷與答案卷是否正確。
(2)測驗時間 110 分鐘。試卷加答案卷、答案卡共計 6 頁。
(3)試卷包括選擇題與填充題，總分共計 100 分，占學期成績之 30%。考卷成績將做為微積分獎給獎依據。
(4)請先確實在答案卡與答案卷填入相關個人資料。答題時請依題號作答，否則不予計分。

◎ 單選擇題　(單選十題，每題五分，共五十分，答錯不倒扣)

1. Which one of the following equation represents the plane that is **tangent** to both of the parametric curves $\boldsymbol{r}_1(s) = \langle s^2+3, -s+1, s\rangle$ and $\boldsymbol{r}_2(t) = \langle t^2, t-2, -t+3\rangle$ at their intersection point?

   (A) $y+z-1=0$;　　(B) $x+z-5=0$;
   (C) $3y+3z-1=0$;　　(D) $x+3y+3z-7=0$.

2. Which one of the following series is **divergent**?

   (A) $\sum_{n=1}^{\infty}\frac{n}{2^n}$;　　(B) $\sum_{n=0}^{\infty}\frac{(-1)^n\sqrt{n}}{n+1}$;
   (C) $\sum_{n=1}^{\infty}\frac{1}{n^2}$;　　(D) $\sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{n}}-\frac{1}{n}\right)$.

3. If $f(x)=xe^x$, then $f^{(100)}(0)=$

   (A) $\frac{1}{99!}$;　　(B) $\frac{1}{100!}$;　　(C) 1;　　(D) 100.

4. Let $f(x,y)=x^2+y^2+x^2y+4$. How many **saddle points** does f possess?

   (A) 0;　　(B) 1;　　(C) 2;　　(D) 3.

5. Let S be the level surface $x\ln y + y\ln z + xyz = 0$ and $\langle a,b,-1\rangle$ be the **normal vector** of the tangent plane to S at the point $(0,1,1)$. Then, the pair (a,b) is equal to

   (A) $(1,0)$;　　(B) $(0,1)$;　　(C) $(-1,0)$;　　(D) $(0,-1)$.

6. The iterated integral $\int_0^1\int_y^1 x^2e^{xy}\,dx\,dy=$

   (A) $\frac{e-2}{2}$;　　(B) $\frac{e-2}{3}$;　　(C) $\frac{e-2}{4}$;　　(D) $\frac{e-2}{5}$.

7. The **surface area** of the part of the surface $z = x + y^2$ that lies above the region $R = \{(x, y) | 0 \leq \mathrm{x} \leq \mathrm{y} \leq 1\}$ is

(A) $\frac{\sqrt{6}}{2}$; (B) $\frac{\sqrt{2}}{6}$; (C) $\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{6}$; (D) $\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{6}$.

8. The iterated integral $\int_0^2 \int_0^{\sqrt{1-(x-1)^2}} \frac{x+y}{x^2+y^2} dydx$ =

(A) $\frac{\pi}{2}$; (B) $\frac{\pi}{4}$; (C) $1 + \frac{\pi}{2}$; (D) $1 + \frac{\pi}{4}$.

9. The triple integral $\iiint_{\mathbb{R}^3} \frac{dV}{(x^2+y^2+z^2+1)^2} =$

(A) π; (B) π^2; (C) π^3; (D) π^4.

10. Let

$$I = \iiint_D f(x, y, z) dV,$$

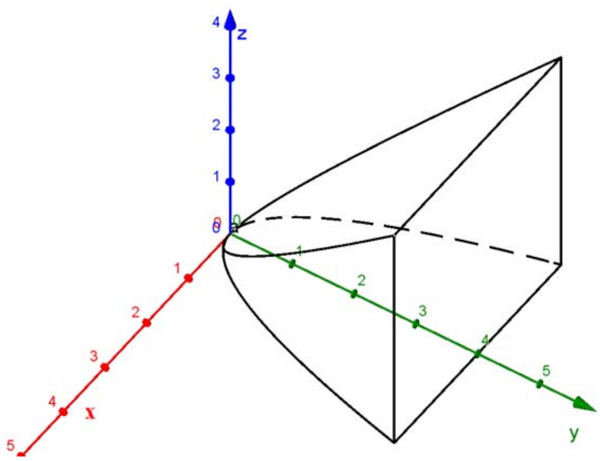
where D is the solid bounded by $y = x^2$, $z = y$ and $y = 4$. Then, for any continuous function f, I always equals

(A) $\int_{-2}^{2} \int_0^4 \int_{x^2}^4 f(x, y, z) dydzdx$;

(B) $\int_{-2}^{2} \int_0^4 \int_z^4 f(x, y, z) dydzdx$;

(C) $\int_{-2}^{2} \int_0^{x^2} \int_{x^2}^4 f(x, y, z) dydzdx + \int_{-2}^{2} \int_{x^2}^4 \int_z^4 f(x, y, z) dydzdx$;

(D) $\int_{-2}^{2} \int_0^{x^2} \int_z^4 f(x, y, z) dydzdx + \int_{-2}^{2} \int_{x^2}^4 \int_{x^2}^4 f(x, y, z) dydzdx$.



◎ 多選擇題 (多選五題，每題五分，共二十五分。答錯一個選項扣兩分，錯兩個選項以上不給分，分數不倒扣)

11. Let $a_n, b_n > 0$ and let $\sum_{n=0}^{\infty} a_n$ be a convergent series. Which of the following statements are **always true**?

(A) If the series $\sum_{n=0}^{\infty} b_n$ converges, then the series $\sum_{n=0}^{\infty} a_n b_n$ also converges;

(B) $\sum_{n=0}^{\infty}(-1)^n a_n$ converges;

(C) If $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$, then the series $\sum_{n=0}^{\infty} b_n$ converges;

(D) $\sum_{n=0}^{\infty} \ln(1 + a_n)$ converges.

12. Which of the following statements are **true**?

(A) If $\left|f^{(k)}(x)\right| \leq 1$ for all $x \in (-1,1)$ and for all positive integers $k \geq 1$, then

$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$ for $x \in (-1,1)$;

(B) There exists a power series $\sum_{n=0}^{\infty} a_n x^n$ whose interval of convergence is $(0, \infty)$;

(C) If the power series $\sum_{n=0}^{\infty} a_n x^n$ converges for $x = 1$, then it also converges for $x = -1$;

(D) If the power series $\sum_{n=0}^{\infty} a_n x^n$ has radius of convergence 2, then $\sum_{n=0}^{\infty} a_n x^{2n}$ has radius of convergence $\sqrt{2}$.

13. Which of the following values of x the series $\sum_{n=0}^{\infty} \frac{nx^n}{(n+1)(2x+1)^n}$ **converges absolutely**?

(A) $x = -1$; (B) $x = -2$; (C) $x = 1$; (D) $x = -\frac{1}{4}$.

14. For the statement: "If the directional derivative $D_{\vec{u}} f(0,0)$ exists for any unit vector $\vec{u}$, then $f(x,y)$ is continuous at the point $(0,0)$", which of the following are **counterexamples**?

(A) $f(x,y) = \begin{cases} \frac{x^2 y}{x^4+y^2}, & \text{if } (x,y) \neq (0,0), \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$

(B) $f(x,y) = \begin{cases} \frac{xy}{x^2+y^2}, & \text{if } (x,y) \neq (0,0), \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$

(C) $f(x,y) = \begin{cases} \frac{(2x^2+x^4)y}{(2x^2+x^4)^2+y^2}, & \text{if } (x,y) \neq (0,0), \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$

(D) $f(x,y) = x^2 + y^2$.

15. Let $F(x,y) = \iint_{D(x,y)} \sin(st)dA$, where $D(x,y) = \{(s,t)|0 \le s \le x, 0 \le t \le 2y\}$. Which of the following are **correct**?

(A) $F_x(x,y) = x^{-1}(1-\cos(xy))$ for $x > 0$ and $y > 0$;

(B) $F_y(x,y) = y^{-1}(1-\cos(2xy))$ for $x > 0$ and $y > 0$;

(C) $F_x(0,0)$ **does not** exist;

(D) $F_{xy} = F_{yx}$ for $x > 0$ and $y > 0$.

◎ 填空題 (五題，每題五分，共二十五分，答錯不倒扣)

1. Suppose that the equation of the **tangent line** to the polar curve $r = 1 + \cos\theta$ at the point $(r,\theta) = (\frac{1}{2}, \frac{2\pi}{3})$ is $ax + by + 1 = 0$. Then the pair (a,b) is ____(1)____.

2. The **area** of the region that lies inside both curves $r = 3\sin\theta$ and $r = 1 + \sin\theta$ is ____(2)____.

3. The **absolute maximum value** of $f(x,y,z) = xy - yz$ subject to the constraint $x^2 + y^2 + \frac{z^2}{2} = 6$ is ____(3)____ .

4. Let $f(x,y) = g\big(u(x,y), v(x,y)\big)$ where $g(u,v) = v\ln(u)$, $u(x,y) = x + y^2$, $v(x,y) = e^y \tan^{-1}(x)$. Suppose that the estimate for $f(1.2, 0.1)$ by using a **linear approximation** at the point $(x,y) = (\mathbf{1}, \mathbf{0})$ is $a\pi$. Then $a =$ ____(4)____.

5. The double integral

$$\iint_R (x-y)^2 \sin^2(x+y)\, dA = \text{\_\_\_\_(5)\_\_\_\_}$$

where R is the parallelogram with successive vertices $(\pi,0), (2\pi,\pi), (\pi,2\pi)$ and $(0,\pi)$.